

MATHEMATICS ENTRANCE EXAM

The test contains 20 questions on 2 pages. Each question is worth 5 points. If you do not wish to choose one of the first five offered answers, you may mark "N", which is worth 0 points. For an incorrect answer, 0.5 points are deducted. If, for a given question, more than one answer is marked or no answer is marked, as well as if the answer is marked incorrectly in any way, 1 point is deducted.

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1. The value of the expression $\frac{\frac{9}{14} \cdot \frac{7}{3} + 4^{-0,5} + (\sqrt{8})^{2/3}}{(0,25)^{-0,5}}$ is equal to:
 A) 4; B) $\frac{1}{3}$; ☒ C) 2; D) 3; E) $\frac{1}{2}$; N) I don't know.
2. First, the price of the painting was reduced by 20%, and then by another 10%, so that it now costs 6480 dinars. The initial price of the painting is equal to:
☒ A) 9000 dinars; B) 9200 dinars; C) 10400 dinars; D) 10000 dinars; E) 12000 dinars; N) I don't know.
3. If for a complex number z it holds that $(2 - i)z = 7 + i$, where $i^2 = -1$, then the value of the expression $\text{Re}(z) - \text{Im}(z)$ is equal to:
☒ A) $\frac{4}{5}$; B) $\frac{22}{5}$; C) 5; D) -5; E) 0; N) I don't know.
4. If $a \neq 0$, $b \neq 0$ and $|a| \neq |b|$, then the expression $\frac{a^2 + ab}{a^2 - ab} : \frac{a^2b + ab^2}{a^2 - b^2}$ is identically equal to:
 A) ab ; B) $\frac{a^2 + b^2}{ab}$; C) $a + b$; ☒ D) $\frac{a + b}{ab}$; E) $\frac{(a + b)^2}{ab}$; N) I don't know.
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5. If $f(x) = \frac{2 - x}{1 + x}$ for $x \neq -1$ and $g(x) = \frac{x}{x + 3}$ for $x \neq -3$, then $g(f(x))$ is equal to:
 A) $\frac{x + 5}{2x - 3}$; ☒ B) $\frac{2 - x}{5 + 2x}$; C) $\frac{3x - 2}{x - 5}$; D) $\frac{2x + 3}{x - 5}$; E) $\frac{2x - 5}{x + 2}$; N) I don't know.
6. The sum of all real solutions of the equation $\frac{4x}{2x + 1} + \frac{2x + 1}{x} = 5$ is equal to:
☒ A) $-\frac{1}{2}$; B) $-\frac{5}{2}$; C) $\frac{3}{2}$; D) 1; E) 0; N) I don't know.
7. If $a = \sqrt[5]{5}^{\log_{25} 4} + \log_3 \frac{1}{\sqrt[5]{81}}$, then $\left(a + \frac{4}{5}\right)^4$ is equal to:
 A) 16; B) 9; C) 2; ☒ D) 4; E) 64; N) I don't know.
8. Given the point $M(5, 8)$ and the circle $(x - 3)^2 + (y - 2)^2 = 4$ centered at point C . The equation of the line containing the midpoint of segment CM and perpendicular to segment CM is:
 A) $x - 2y = 7$; B) $3x - y = 7$; ☒ C) $x + 3y = 19$; D) $3x - y = -7$; E) $3x + y = 19$; N) I don't know.
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9. All real solutions of the equation $4 \cdot 3^x + 24 \cdot 3^{x-1} - 8 \cdot 3^{x+1} = -36$ belong to the interval:
☒ (A) $(0, 1]$; B) $(2, 4]$; C) $(1, 2]$; D) $(4, +\infty)$; E) $(-\infty, 0]$; N) I don't know.
10. For an arithmetic progression $a_1, a_2, \dots$, it holds that $a_1 = 11$ and $a_7 = 35$, and for an arithmetic progression $b_1, b_2, \dots$, it holds that $b_1 = 38$ and $b_n = 13$. If $a_4 = b_4$, then n is equal to:
A) 12; B) 5; ☒ C) 6; D) 10; E) 7; N) I don't know.
11. The number of all integer solutions of the inequality $\sqrt{(x-6)(1-x)} < 3 + 2x$ is equal to:
A) 7; B) 4; C) 3; D) 5; ☒ E) 6; N) I don't know.
12. The largest real solution of the inequality $\log_7(1-2x) \geq \log_7(5x-2)$ is equal to:
A) $\frac{5}{11}$; B) $\frac{4}{9}$; C) $\frac{7}{15}$; D) $\frac{6}{13}$; ☒ E) $\frac{3}{7}$; N) I don't know.
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13. If both the polynomial $ax^3 - bx^2 - bx + a$ and the polynomial $bx^3 - 5x^2 - 2ax + 2a$ are divisible by the polynomial $x - 2$, where $a, b \in \mathbb{R}$, then $a + b$ is equal to:
A) 6; B) 3; ☒ C) 5; D) 9; E) 7; N) I don't know.
14. The value of the expression $\cos 15^\circ$ is equal to:
A) $\frac{\sqrt{6} - \sqrt{2}}{2}$; ☒ B) $\frac{\sqrt{2 + \sqrt{3}}}{2}$; C) $\frac{\sqrt{3}}{4}$; D) $\frac{\sqrt{\sqrt{6} - \sqrt{2}}}{4}$; E) $\frac{\sqrt{3}}{8}$; N) I don't know.
15. A circle is inscribed in a right triangle, and its point of tangency with the hypotenuse divides the hypotenuse into segments of lengths 5 cm and 12 cm. The sum of the lengths of the legs of that right triangle is equal to:
A) 35 cm; B) 27 cm; ☒ C) 23 cm; D) 34 cm; E) 20 cm; N) I don't know.
16. If the ratio of the surface area to the volume of a regular tetrahedron with edge length a is equal to $2\sqrt{3}$, then the ratio of the surface area to the volume of a right circular cone whose base radius and height both have length a is equal to:
☒ A) $\frac{2 + \sqrt{2}}{2}$; B) $\frac{1}{2}$; C) $\sqrt{2}$; D) $\frac{\sqrt{2}}{3}$; E) $\frac{1}{\sqrt{2}}$; N) I don't know.
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17. The number of all solutions of the equation $\sin 2x = (\cos x - \sin x)^2$ that belong to the interval $[0, 2\pi]$ is equal to:
A) 5; B) 3; C) 8; ☒ D) 4; E) 6; N) I don't know.
18. A laboratory employs 20 people, exactly one of whom is the laboratory manager, exactly one is the deputy manager, and exactly one is the chief engineer. One employee can hold at most one of these three positions. The number of ways in which the laboratory can send 5 employees on a business trip, given that the laboratory manager, the deputy manager, and the chief engineer cannot all be absent at the same time, is equal to:
A) 11404; B) 16806; C) 15658; D) 11224; ☒ E) 15368; N) I don't know.
19. The maximum volume of a right circular cone whose slant height has length 3 cm is equal to:
A) $3\pi \text{ cm}^3$; ☒ B) $2\sqrt{3}\pi \text{ cm}^3$; C) $4\pi \text{ cm}^3$; D) $\sqrt{3}\pi \text{ cm}^3$; E) $2\pi \text{ cm}^3$; N) I don't know.
20. The number of terms in the expansion of $\left(\sqrt[3]{2} + \sqrt[4]{8}\right)^{50}$ that are integers is equal to:
A) 5; B) 7; C) 8; D) 6; ☒ E) 4; N) I don't know.
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