

MATHEMATICS ENTRANCE EXAM

The test contains 20 questions on 2 pages. Each question is worth 5 points. If you do not wish to choose one of the first five offered answers, you may mark "N", which is worth 0 points. For an incorrect answer, 0.5 points are deducted. If, for a given question, more than one answer is marked or no answer is marked, as well as if the answer is marked incorrectly in any way, 1 point is deducted.

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1. The value of the expression $(0,5)^{-2} + (0,2)^{-2} - (-0,125)^{-2} \cdot \sqrt{(-0,5)^4}$ is equal to:
A) 11; B) 13; C) 8; D) 15; E) 10; N) I don't know.
2. The price of a book was first increased by 40%, and then decreased by 40%, such that now the price is equal to 2520 dinars. The initial price was equal to:
A) 2116 dinars; B) 3528 dinars; C) 2520 dinars; D) 1008 dinars; E) 3000 dinars; N) I don't know.
3. If $z = \frac{i^{2025} + i^{2026}}{i^{2021} + i^{2024}}$, where $i^2 = -1$, then z^2 is equal to:
A) 2; B) $-2i$; C) $2i$; D) -2 ; E) -1 ; N) I don't know.
4. If $p \neq 0$, $q \neq 0$ and $|p| \neq |q|$, then the expression $\left(\left(\frac{p^3}{q} - \frac{q^3}{p}\right) : \left(\frac{p}{q} + \frac{q}{p}\right)\right)^{-1} \cdot (p - q)$ is identically equal to:
A) pq ; B) $p + q$; C) $\frac{1}{p + q}$; D) $\frac{1}{pq}$; E) $\frac{1}{p - q}$; N) I don't know.
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5. If $f\left(\frac{1-x}{1+x}\right) = \frac{1}{x}$ for $x \notin \{-1, 0\}$, and if f^{-1} is the inverse function of f , then the value of $f(2) \cdot f^{-1}(2)$ is equal to:
A) -3 ; B) -1 ; C) 9; D) 1; E) 3; N) I don't know.
6. The number of all integer solutions of the inequality $\frac{(x-3)(x^2-8x+15)}{x+2} \leq 0$ is equal to:
A) 3; B) 5; C) 2; D) 7; E) 1; N) I don't know.
7. The value of the expression $2^{\log_4 25} - 7^{1 + \log_7(2/7)}$ is equal to:
A) 2; B) 6; C) 5; D) 4; E) 3; N) I don't know.
8. If points $A(1, 2)$ and $B(3, 4)$ lie on the circle k , whose center C lies on the line $y = 2x - 4$, then the perimeter of the triangle ABC is equal to:
A) $4\sqrt{2}$; B) $2(2 + \sqrt{2})$; C) $2(1 + 2\sqrt{2})$; D) 5; E) 7; N) I don't know.
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9. The smallest real solution of the equation $3 \cdot 4^{x+1} + 2 \cdot 9^{x+1} = 35 \cdot 6^x$ belongs to the interval:
A) $[-3, -2)$; B) $[-4, -3)$; C) $[-5, -4)$; D) $[-2, -1)$; E) $[-1, 0)$; N) I don't know.
10. The common difference d of the arithmetic progression a_1, a_2, \dots is an integer, and the sum of first ten terms of that progression is 155. The common ratio q of the geometric progression b_1, b_2, \dots is also an integer, and the sum of first two terms of that progression is 9. If $a_1 = q$ and $b_1 = d$, then b_4 is equal to:
 A) 24; B) 96; C) 48; D) 54; E) 64; N) I don't know.
11. The product of all real solutions of the equation $\sqrt{2x+8} - \sqrt{x+2} = 2$ is equal to:
 A) -28; B) -4; C) 28; D) -14; E) 14; N) I don't know.
12. The number of all positive solutions of the equation $x + \log_{10}(2^x + 2) = \log_{10}(3 \cdot 5^{x+1})$ is equal to:
 A) 1; B) 4; C) 0; D) 2; E) 3; N) I don't know.
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13. If the polynomial $P(x) = x^{2026} + ax^{2025} + bx^2 + 3x - 1$, $a, b \in \mathbb{R}$, is divisible by the polynomial $Q(x) = x^2 - \sqrt{2}x + 1$, then the value of the expression $(2a - 3b)^2$ is equal to:
 A) 8; B) 6; C) $4\sqrt{2}$; D) $\sqrt{2}$; E) $6\sqrt{2}$; N) I don't know.
14. The value of the expression $\operatorname{tg} 1^\circ \cdot \operatorname{tg} 2^\circ \cdot \operatorname{tg} 3^\circ \cdot \dots \cdot \operatorname{tg} 88^\circ \cdot \operatorname{tg} 89^\circ$ (i.e. $\prod_{k=1}^{89} \operatorname{tg} k^\circ$) is equal to:
A) -1; B) $\frac{1}{290}$; C) $\frac{1}{289}$; D) 1; E) $\frac{1}{245}$; N) I don't know.
15. The length of the hypotenuse of a right-angled triangle ABC is equal to 40 cm. The perpendicular bisector of the hypotenuse intersects the hypotenuse at point S , and the longer leg of the triangle ABC at point M . If the length of the line segment SM is 15 cm, then the sum of the lengths of the legs of the triangle ABC is equal to:
 A) 56 cm; B) 54 cm; C) 52 cm; D) 48 cm; E) 50 cm; N) I don't know.
16. A right regular hexagonal prism is given, such that its height has the same length as the side of its base. First, a ball is circumscribed around the given prism, and then a cone is inscribed inside the prism, such that the base of the cone is tangent to the sides of one base of the prism, and the vertex of the cone is the center of the circle inscribed in the other base of the prism. The ratio of volumes of the circumscribed sphere and the inscribed cone is equal to:
A) $15\sqrt{5} : 8$; B) $4\sqrt{3} : 1$; C) $10\sqrt{5} : 6$; D) $10\sqrt{5} : 3$; E) $4 : \sqrt{3}$; N) I don't know.
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17. The number of all solutions of the equation $\sin(2026x) + \sin(2024x) = -2 \cos x$ which are elements of the interval $(0, 2\pi]$ is equal to:
A) 2023; B) 2024; C) 2026; D) 2025; E) 2027; N) I don't know.
18. The number of all odd four-digit numbers whose digits are all mutually different is equal to:
A) 2560; B) 2240; C) 2122; D) 2880; E) 2520; N) I don't know.
19. The minimal sum of distances from arbitrary point on the line $x = 1$ to points $M(4, -2)$ and $N(7, 10)$ is equal to:
A) $\frac{31}{2}$; B) $\frac{33}{2}$; C) $\frac{27}{2}$; D) 13; E) 15; N) I don't know.
20. If the number of terms in the expansion $(\sqrt{2} + \sqrt[3]{4})^n$ which are integers is 50, and the number of all terms is divisible by 10, then the sum of all binomial coefficients of this expansion is equal to:
A) 4^{150} ; B) 2^{301} ; C) 2^{299} ; D) 2^{151} ; E) 2^{149} ; N) I don't know.
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